Mathematics 263 Ordinary Differential Equations and Linear Algebra

1. Find the general solutions to the following ODE's; in part (a), also find the specific solution satisfying the given initial condition.

   (a) $(2yx - 2x)y' = 3y^2 - 6y$ with $y(0) = 2$.

   (b) $(1 + x^2)(y' + 3y) = e^{-3x}$

   (c) $(15x^2 + 8xy^2 - 18y)dx + (4x^2y - 6x)dy = 0$

   (d) $xy' + xy = e^{2x}y^3$

2. Solve the following Euler equation. $2x^2y'' + 3xy' - y = x^{-\frac{1}{2}}$ $(x > 0)$.

3. Using variation of parameters, solve $y'' + 4y = \dfrac{1}{\cos 2x}$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

4. For the ODE $x^2y'' - (3x + 2x^3)y' + (3 + 2x^2)y = 0$, verify that $y_1 = x$ is a solution. Find the general solution of the given ODE.

5. Solve the following initial value problem. $(D^3 - D)y = x + 3e^x$ with $y(0) = y'(0) = y''(0) = 0$.

6. Find the inverse Laplace transforms of the following

   (a) $F(s) = \dfrac{1}{(s^4 - 1)}$

   (b) $F(s) = \dfrac{s}{(s^2 + 1)^2}$

   (c) $F(s) = \dfrac{s - 3}{s^2 + 4s + 5}$

7. Solve $y' - 2y = f(t) + \delta(t - 2)$ with $y(0) = 0$ where $f(t)$ is given by

$$f(t) = \begin{cases} 2t & 0 \le t < 1 \\ 2 & 1 \le t \end{cases}$$

8. Put the following system into matrix form $D\mathbf{v} = A\mathbf{v}$ where $\mathbf{v} = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$

$$\begin{aligned} y_1' &= 4y_1 - 3y_2 \\ y_2' &= 2y_1 - 3y_2 \end{aligned}$$

   (a) Display the matrix $A$ and find a basis of $\mathcal{R}^2$ consisting of eigenvectors of $A$.

   (b) Find the general solution of the system $D\mathbf{v} = A\mathbf{v}$.

   (c) Calculate $e^{At}$.

9. Find bases for the column space, null space (i.e. solutions of the corresponding homogeneous equations) and row space for the following matrix $A$

$$A = \begin{pmatrix} 1 & -2 & 3 & 2 & 0 \\ -3 & 6 & -8 & 5 & -2 \\ 4 & -8 & 11 & -3 & 2 \end{pmatrix}$$